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STRAIN CHARACTERISTICS OF MATERIALS WITH DEFECTS

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For uniaxial tension of St. 3 *steel, the stiffness-matrix components are determined using a two-dimensional plastic solid model and taking into account the formation and development of anisotropic damage. In order to characterize defect formation, a vector damage criterion is introduced, mictrostructural analysis data are used, and the destruction point is established. Estimates of the elastic moduli and experimental data indicate that the model provides a qualitative description of the real deformation and failure processes of deforming and destruction and can be used to determine the life of materials.*

Key words: *plastic solid, slip systems, elastic moduli, anisotropy, defects.*

To construct strength theory and describe the fracture of solids, it is first necessary to identify the real degree of imperfection and take into account its effect on the strain characteristics of materials. It is clear that this inevitably complicates the present models of mechanics, which should be based on experimental data and physical mechanisms of the fracture process.

In both the analytical apparatus of the theory of deformable solids and engineering developments, the main parameters are elastic constants or elastic moduli. Experimental data have shown that these quantities are structurally sensitive characteristics that depend not only on the chemical composition but also on the imperfection and isotropy of the material. In particular, the dependence of the elastic modulus on damage underlies one of the methods for determining the imperfection of solids under the assumption that the elastic modulus of a damaged medium is equal to a certain effective modulus of the undamaged continuum and the presence of defects is characterized by the scalar parameter D [1]. However, under loading, defects with preferential orientation develop and the initially isotropic solid becomes anisotropic with orthotropic symmetry [2].

The present paper deals with determining the elastic characteristics of plastically deformed materials taking into account the formation and development of anisotropic damage. The problem is solved using the Batdorf–Budiansky two-dimensional model of a plastic solid [3] and experimental data on the elastic modulus for unloading.

1. We consider the case of uniaxial loading of a plastic solid taking into account the formation and growth of microcracks. For definiteness, we assume that the growth of microcracks is only due to the Stroh plastic mechanism [4] (this process was studied analytically in [5, 6]) and that the solid, as in [3], consists of an aggregate of grains which have a single slip system determined by the mutually orthogonal directions $\boldsymbol{n}$ and $\boldsymbol{\lambda}$. A shear stress $\tau_{n\lambda}$ acts in the slip system. If the grains are large in number, among them there are grains for which the normal to the slip plane is inside a solid angle $d\Omega$ with axis $\boldsymbol{n}$ and the slip direction is inside an angle $d\lambda$ with bisectrix λ. Thus, the number of grains that have the slip system $n\lambda$ is proportional to $d\Omega\, d\lambda$. From this, for the deformation of the continual medium and the two-dimensional solid model we assume that:

— The plastic strain of the solid is the sum of the irreversible shears in the slip planes $n\lambda$ that are perpendicular to the plane of load application xy;

— Irreversible shears occur only in those planes in which there is at least one direction along which $\tau_{n\lambda}$ exceeds the critical constant value and, in addition, is larger than its previous values;

— The value of the plastic shear $\gamma_{n\lambda}$ depends only on $\tau_{n\lambda}$;

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— The slip systems do not interact with each other and the total strain is summed over all directions;

— When the stress–strain state reaches the condition of microcrack initiation, which reduces to satisfying the condition $\tau_{n\lambda} = \tau_s$, k_0 defects of size l_0 form in the slip system $n\lambda$, which represent a periodic system of slits with distance a between the centers of neighboring microcracks;

— During further deformation, the remaining intact part of the slip system is deformed by a shear whose value depends only on the effective shear stress;

— The variation in the microcrack size is determined by the condition of microcrack growth, which is written as [7]

$$\frac{l_k - l_k^0}{a} = B(\gamma_k - \gamma_k^0)\sinh\frac{\sigma_0}{\sigma_i}, \tag{1}$$

where l_k is the microcrack length in the kth slip system, γ_k is the shear plastic strain in the kth slip system, γ_k^0 is the shear plastic strain that leads to generation of microcracks of size l_k^0 in the kth slip system, σ_0 is the spherical part of the stress tensor, σ_i is the strain intensity, and B is a constant,

— For unloading before the moment of microcrack formation, we have an elastic isotropic solid with elastic constants E, ν, and $G = E/(2+2\nu)$ in the coordinate system $\boldsymbol{\rho q}$ attached to the proportional strain path. In this case, the stress–strain state of an orthotropic medium with damage can be described by a potential of quadratic form [8] whose variables are the strain-tensor components ε_j and the damage-vector components ω_j:

$$\begin{aligned} W = {} & k_1\varepsilon_1^2 + k_2\varepsilon_1\varepsilon_2 + k_3\varepsilon_1^2\omega_1^2 + k_4\varepsilon_1^2\omega_2^2 + k_5\varepsilon_1\varepsilon_6\omega_1\omega_2 + k_6\varepsilon_2^2 + k_7\varepsilon_2^2\omega_1^2 + k_8\varepsilon_2^2\omega_2^2 \\ & + k_9\varepsilon_2\varepsilon_6\omega_1\omega_2 + k_{10}\varepsilon_6^2 + k_{11}\varepsilon_6^2\omega_1^2 + k_{12}\varepsilon_6^2\omega_2^2 + k_{13}\varepsilon_1\varepsilon_2\omega_1^2 + k_{14}\varepsilon_1\varepsilon_2\omega_2^2 + P. \end{aligned} \tag{2}$$

Here k_i are the decomposition coefficients and P is a polynomial quadratic in ω_j.

In the case of microcrack formation in the slip system attached to the loading direction, microcracks form in the direction $\boldsymbol{q}$ and the coordinates of the vector $\boldsymbol{\omega}$ are $(0, \omega_2)$. Hence, in a coordinate system attached to the kth slip plane in which axis 1 is directed along the slip axis and axis 2 along the normal to the slip axis, using (2), we obtain the following expressions for the elastic moduli:

$$\begin{gathered} C_{11} = C_{11}^0 + 2k_4\omega_2^2, \qquad C_{12} = C_{21} = C_{12}^0 + k_{14}\omega_2^2, \qquad C_{22} = C_{22}^0 + 2k_8\omega_2^2, \\ C_{66} = C_{66}^0 + 2k_{12}\omega_2^2, \qquad C_{16} = C_{61} = 0, \qquad C_{26} = C_{62} = 0, \end{gathered} \tag{3}$$

where C_{ij}^0 are the elastic constants of the undamaged slip systems:

$$C_{11}^0 = C_{22}^0 = E/(1-\nu^2), \qquad C_{66}^0 = G = E/(2+2\nu), \qquad C_{12}^0 = \nu E/(1-\nu^2). \tag{4}$$

In the coordinate system attached to the proportional loading path and the unit bases $\boldsymbol{\rho}$ and $\boldsymbol{q}$, the elastic constants $\bar{C}_{ij}^k$, according to the transformation rules [9], are defined by

$$\bar{C}_{ij}^k = a_{ijl}^k U_l^k, \qquad l = 1, 2, 3, 4, \tag{5}$$

where

$$\begin{gathered} U_1^k = (1/8)(3C_{11} + 3C_{22} + 2C_{12} + 4C_{66}), \qquad U_2^k = (1/2)(C_{11} - C_{22}), \\ U_3^k = (1/8)(C_{11} + C_{22} - 2C_{12} - 4C_{66}), \qquad U_4^k = (1/8)(C_{11} + C_{22} + 6C_{12} - 4C_{66}), \end{gathered}$$

a_{ijl}^k are orientation coefficients.

Using (3), we obtain

$$U_l^k = U_l^{0k} + A_l^k\omega_2^2, \tag{6}$$

where A_l^k are expressed in terms of the coefficients k_i of expansion of the strain potential (2):

$$\begin{gathered} A_1 = (1/4)(3k_4 + 3k_8 + 4k_{12} + k_{14}), \qquad A_2 = k_4 - k_8, \\ A_3 = (1/4)(k_4 + k_8 - 4k_{12} - k_{14}), \qquad A_4 = (1/4)(k_4 + k_8 - 4k_{12} + 3k_{14}). \end{gathered} \tag{7}$$

The stress tensor components in the solid are obtained by summation over all slip planes, namely, by integration over the angle λ from $-\pi/2$ to $\pi/2$ taking into account (5) and (6):

$$\sigma_i = \frac{1}{\pi} \int\limits_{-\pi/2}^{\pi/2} \bar{C}_{ij}^k \, d\lambda \varepsilon_j = \frac{1}{\pi} \int\limits_{-\pi/2}^{\pi/2} a_{ijl}^k U_l^{0k} \, d\lambda \varepsilon_j + \frac{1}{\pi} \int\limits_{-\pi/2}^{\pi/2} a_{ijl}^k A_l \omega_2^{k^2} \, d\lambda \varepsilon_j. \qquad (8)$$

The first integral in (8) gives the stiffness matrix of linear isotropic elastic theory, and in the second integral the integrand is nonzero for $\lambda \in [-\varphi, \varphi]$ because in the range $-\pi + \varphi < \lambda < \pi - \varphi$, where the angle φ is determined from the condition $\cos\varphi = \tau_s/\tau$ [10], the material is undamaged and $\omega_2 = 0$ in this case. The constants A_l are factored outside the integral sign and $\omega_2^{k^2}$ is a positive symmetric function of λ with respect to the value $\lambda = 0$ for proportional loading. In the kth slip system, ω_2 is a function of the microcrack length l, which, in turn, depends on the shear plastic strain γ^p in this slip system and is determined by the microcrack growth conditions. Then, assuming that $\omega_2^{k^2} = l_k/a$, taking into account that $\gamma = e\cos\lambda/\sqrt{2}$ and $\sigma_0/\sigma_i = \text{const}$, and using the crack growth condition (1), we obtain

$$\sigma_i = [C_{ij}^0]\varepsilon_j + A_l \frac{1}{\pi} \int\limits_{-\varphi}^{\varphi} a_{ijl}^k B_1(e - e_0)\varepsilon_j \cos\lambda \, d\lambda + A_l \frac{1}{\pi} \int\limits_{-\varphi}^{\varphi} a_{ijl}^k \frac{l_k^0}{a} \varepsilon_j \, d\lambda = [C_{ij}^0]\varepsilon_j + [Z_{ij}]\varepsilon_j,$$

where e is the strain intensity, e_0 is the strain intensity for crack initiation, B_1 is a constant, $1/\pi$ is the normalization coefficient, and the matrix has the form

$$[Z_{ij}] = \frac{1}{\pi}\Big([N_{ij}]B_1(e - e_0) + [M_{ij}] \frac{l_k^0}{a}\Big). \qquad (9)$$

Accordingly, the matrices

$$[N_{ij}] = A_l \int\limits_{-\varphi}^{\varphi} a_{ijl}^k \cos\lambda \, d\lambda, \qquad [M_{ij}] = A_l \int\limits_{-\varphi}^{\varphi} a_{ijl}^k \, d\lambda$$

are easily calculated taking into account the expressions for A_l and a_{ijl}^k:

$$\begin{gathered} N_{11} = 2A_1 \sin\varphi + A_2(\sin\varphi + (1/3)\sin 3\varphi) + A_3((1/3)\sin 3\varphi + (1/5)\sin 5\varphi), \\ M_{11} = 2A_1\varphi + A_2 \sin 2\varphi + (1/2)A_3 \sin 4\varphi, \\ N_{22} = 2A_1 \sin\varphi - A_2(\sin\varphi + (1/3)\sin 3\varphi) + A_3((1/3)\sin 3\varphi + (1/5)\sin 5\varphi), \\ M_{22} = 2A_1\varphi - A_2 \sin 2\varphi + (1/3)A_3 \sin 4\varphi, \\ N_{12} = 2A_4 \sin\varphi - A_3((1/3)\sin 3\varphi + (1/5)\sin 5\varphi), \quad M_{12} = 2A_4\varphi - (1/2)A_3 \sin 4\varphi, \\ N_{66} = A_1 \sin\varphi - A_3((1/3)\sin 3\varphi + (1/5)\sin 5\varphi) - A_4 \sin\varphi, \quad M_{66} = A_1\varphi - (1/2)A_3 \sin 4\varphi - A_4\varphi. \end{gathered} \qquad (10)$$

It is obvious that the damaged medium is anisotropic and its stiffness is determined by the coefficients A_l.

2. Let us estimate the coefficients k_i using (3) for materials whose plastic deformation was studied by microstructural analysis in [6, 11]. We assume that at the moment of defect nucleation, a shear occurs in one slip system, the solid retains isotropy, and the elastic modulus of the damaged material is defined by the expression [6]

$$E = \frac{2E_0(1 - c)(7 - 5\nu_0)}{2(7 - 5\nu_0) + (1 + \nu_0)(13 - 15\nu_0)c},$$

where c is the defect concentration. The initial values of the damage parameter and the defect concentration are found by calculating the volume of a conditional pore and the volume per one pore; then,

$$\omega_0 = l_0 N^{-1/3}, \qquad c_0 = \pi l_0^3 N/6,$$

where N is the number of defects in 1 m^3 and l_0 is the size of the microdefect nucleation center determined by microstructural studies. The initial and calculated defect characteristics and estimates of k_i are listed in Table 1. As is evident from the table, the obtained values of k_i are negative, leading to a decrease in all elastic moduli during defect formation with the order of magnitude dependent on the defect concentration and size.

TABLE 1

Material	l_0, μm	c_0	ω_0	k_4, MPa	k_{12}, MPa	k_{14}, MPa
Aluminum	0.14	$1.4 \cdot 10^{-4}$	0.0650	−199.1	−49.8	−199.1
Copper	0.25	$4.1 \cdot 10^{-3}$	0.1984	−2060.5	−515.1	−2060.5
St. 3	0.10	$5.2 \cdot 10^{-7}$	0.0100	−9.2	−2.5	−8.4
VT-5 titanium alloy	3.00	$5.0 \cdot 10^{-3}$	0.0295	−23 700	−5900	−24 800

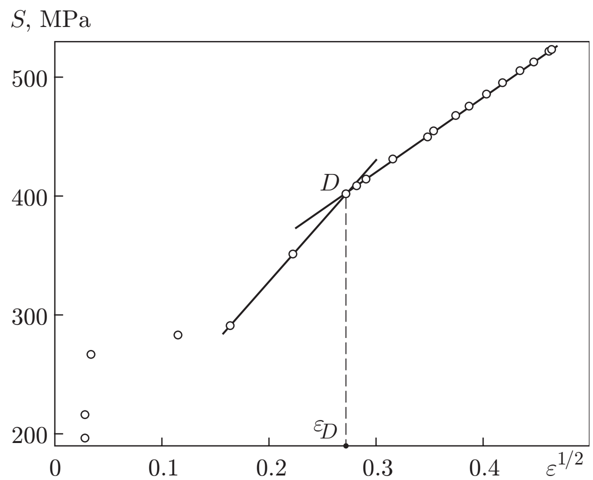

Fig. 1. Destructive loading diagram for St. 3 samples (S is the true stress and ε is the permanent strain): the points refer to experiments and the solid curves are an approximation.

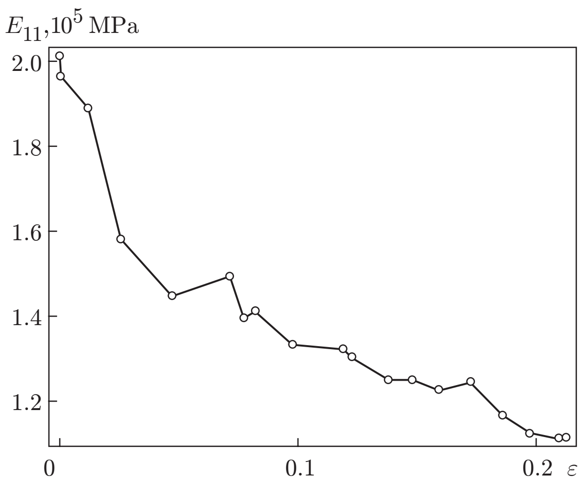

Fig. 2

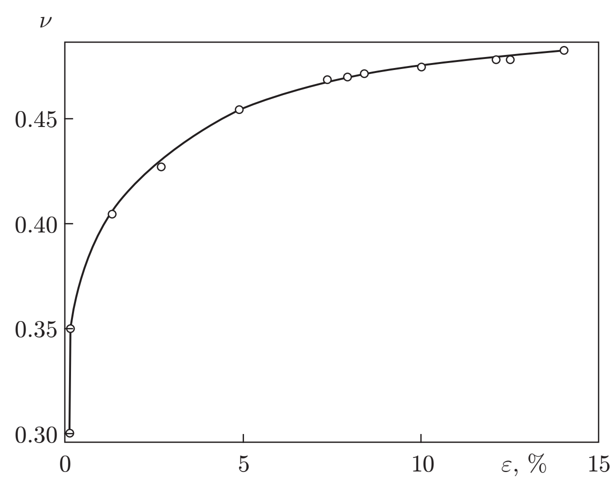

Fig. 3

Fig. 2. Elastic modulus versus permanent strain for St. 3 samples.

Fig. 3. Transverse-strain coefficient versus permanent strain for St. 3 samples.

To determine the effect of anisotropy due to damage accumulation, we consider the behavior of C_{ij} for the deformation of St. 3 steel samples subjected to uniaxial tension with recording of longitudinal and transverse strains.

It is known that in plastically deformed metals, the moment of microdefect nucleation is characterized by the destruction point D, which can be established by various methods [12, 13]. Figures 1–3 show a destructive loading diagram, the variation in the elastic modulus E_D during unloading, and the dependence of the coefficient ν on plastic strain. As follows from Fig. 1, the moment of microdefect nucleation corresponds to a permanent strain $\varepsilon_{11} \approx 0.07$ and a modulus $E_{11} \approx 1.5 \cdot 10^5$ MPa. Then, to find the unknown coefficients A_l and B_1, we formulate

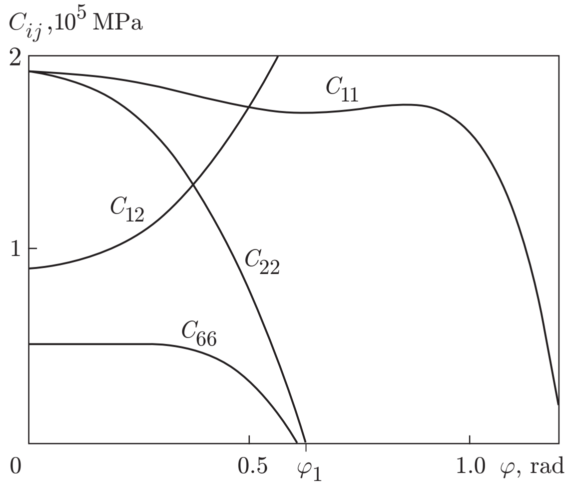

Fig 4

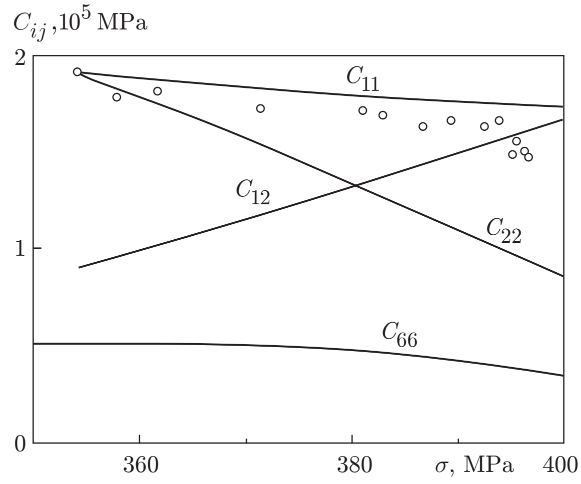

Fig. 5

Fig. 4. Stiffness-matrix components versus slip direction.

Fig. 5. Stiffness-matrix components versus applied stress.

the following system of equations using (9) and experimental data:

$$C_{11}^l = C_{11}^0 + (1/\pi)B_1(e - e_0)N_{11}^l + (1/\pi)\omega_0 M_{11}^l,$$
$$C_{66}^1 = C_{66}^0 + (1/\pi)B_1(e - e_0)N_{66}^1 + (1/\pi)\omega_0 M_{66}^1. \quad (11)$$

To determine C_{ij}^l, we again employ formulas (4) under the assumption that the strain intensity is defined by the relation

$$e = (2\sqrt{2}/3)(1 + \nu + \nu^2)^{1/2}\varepsilon_{11}.$$

Solution of system (11) using (10) for the examined St. 3 steel sample and the indicated assumptions gives the following estimates: $A_1 \approx -16.6 \cdot 10^5$ MPa, $A_2 \approx 24.5 \cdot 10^5$ MPa, $A_3 \approx -26.1 \cdot 10^5$ MPa, $A_4 \approx 24.1 \cdot 10^5$ MPa, and $B_1 \approx 5$. From this, taking into account (7), we have $k_4 \approx -9.2 \cdot 10^5$ MPa, $k_8 \approx -34.9 \cdot 10^5$ MPa, $k_{12} \approx 2.9 \cdot 10^5$ MPa, and $k_{14} \approx 50.7 \cdot 10^5$ MPa.

Unlike in the case of isotropy, the stiffness matrix is now characterized by two negative coefficients k_4 and k_8 and two positive coefficients k_{12} and k_{14}, which indicates the opposite behavior of the elastic moduli. Figure 4 gives a distribution of the values of C_{ij} over the slip systems determined by the angle φ. Generally, with increase in damage, the values of C_{11} decrease and those of C_{12} increase; in this case, C_{22} and C_{66} decrease to zero approximately two times faster than C_{11}. From the results it follows that on reaching a certain stress state that corresponds to the angle φ_1, the plastic deformation becomes unstable. In this case again, a local shear occurs in the plane $\lambda = 0$, and in the planes where $\lambda \neq 0$, plastic shears do not occur. Ultimately, this leads to a decrease in the material strength in the direction C_{11} and fragmentation of the material. The strain at which C_{11} vanishes corresponds to failure of the sample.

Indeed, physical studies have shown that in materials similar to St. 3, plastic deformation occurs by shear over the slip planes of individual ferrite grains in the direction of the major diagonal. The different orientation of the grains and the presence of grain boundaries and inclusions hinder the general shear of one part of the sample relative to the other. Therefore, for common shear planes to form in the sample, the shears in individual ferrite grains should flow around stronger perlite grains or cleave their weak segments with increase in the stress. Figure 5 gives calculated and experimental curves of the elastic moduli versus stress. As is evident, the values of C_{11} determined theoretically and experimentally are close. The small difference is due to the adopted assumptions in the conversion of the measured values of E_{11}.

Thus, an estimate was obtained for the stiffness matrix component in the case of development of anisotropy of the material. The crack orientation is shown to have a significant effect on the behavior of the elastic moduli. The model proposed for the deformation and failure of elastoplastic materials adequately describes the process and can be used as a basis for monitoring the damage level of a materials and for estimating its life.

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